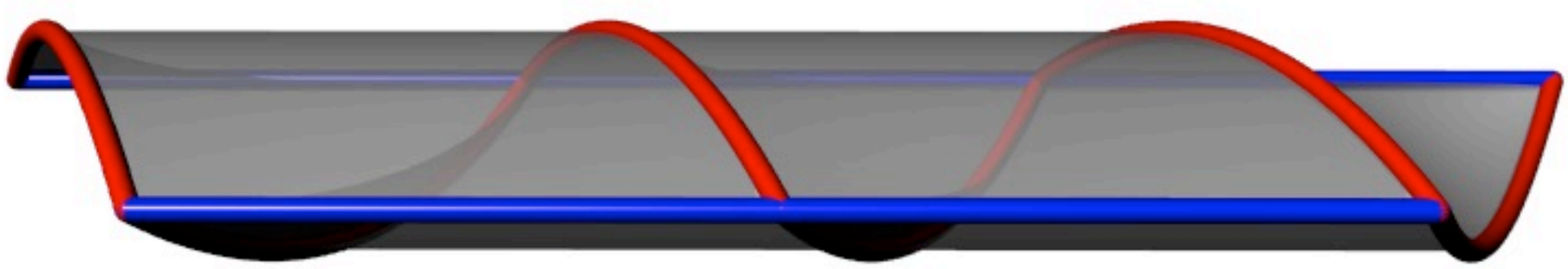
The veering census

Saul Schleimer
University of Warwick
ICERM 2019-11-03

joint work with
Henry Segerman



Illustrating Dynamics and Probability

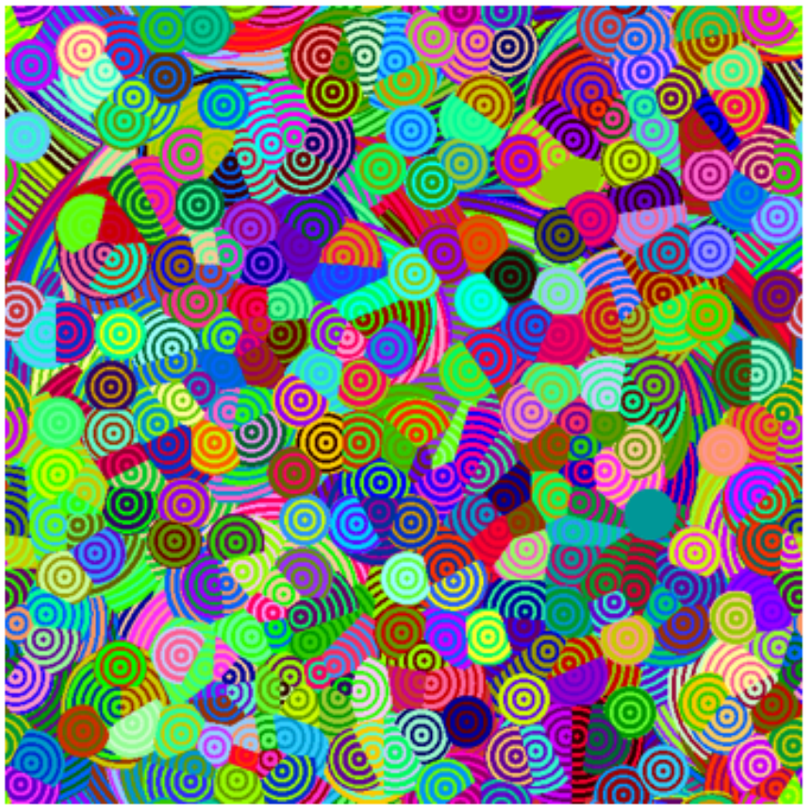
Nov 11 - 15, 2019

Organizing Committee

- Jayadev Athreya
  University of Washington
- Alexander Holroyd
  Churchill College and Statistical Laboratory at University of Cambridge
- Sarah Koch
  University of Michigan, Ann Arbor

Abstract

This workshop will focus on the theoretical insights developed via illustration, visualization, and computational experiment in dynamical systems and probability theory. Some topics from complex dynamics include: dynamical moduli spaces and their dynamically-defined subvarieties, degenerations of dynamical systems as one moves toward the boundary of moduli space, and the structure of algebraic data coming from a family of dynamical systems. In classical dynamical systems, some topics include: flows on hyperbolic spaces and Lorentz attractors, simple physical systems like billiards in two and three dimensional domains, and flows on moduli spaces. In probability theory, the workshop features: random walks and continuous time random processes like Brownian motion, SLE, and scaling limits of discrete systems.



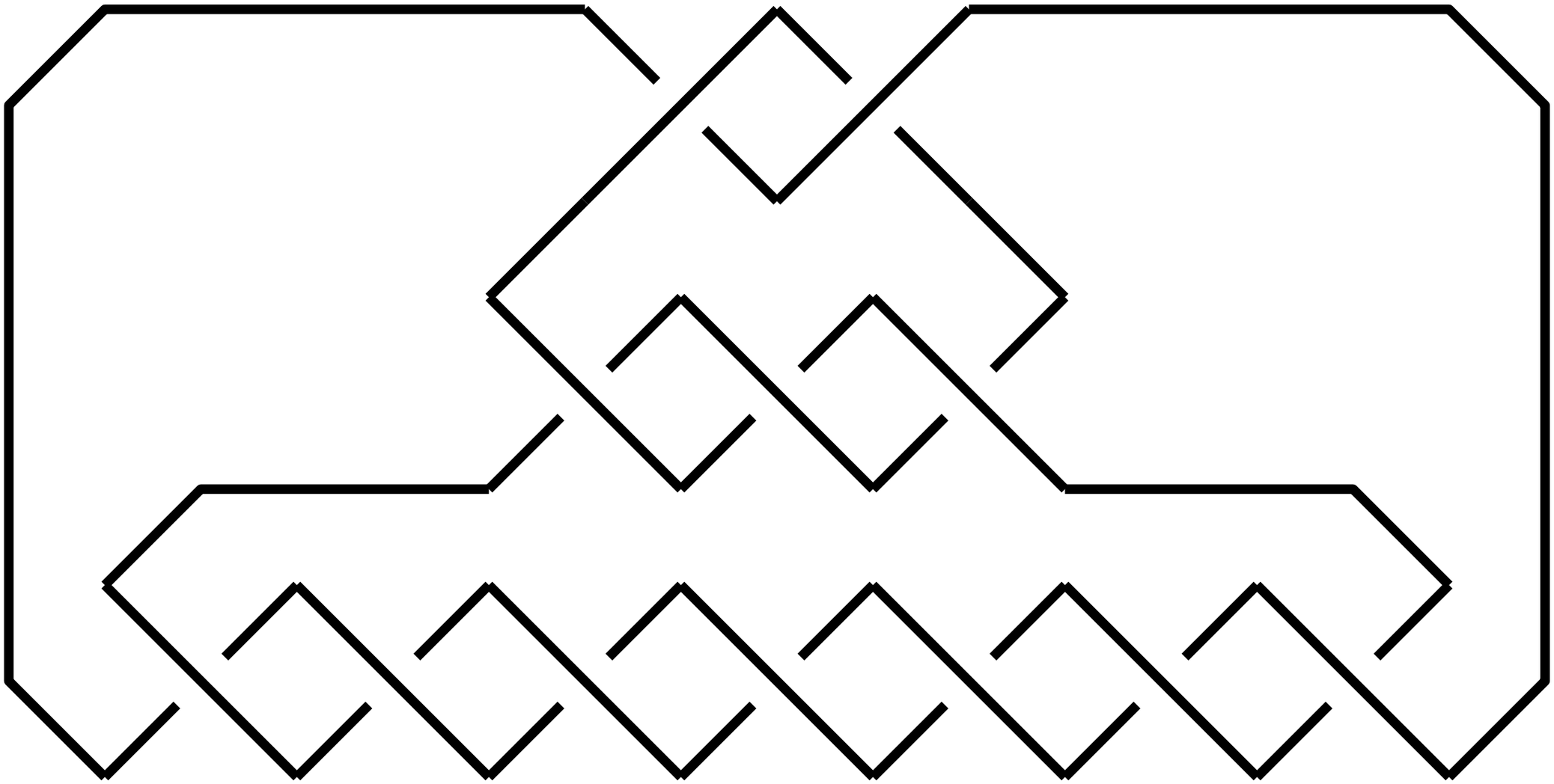
A stable matching in the plane.
Image credit: Alexander E. Holroyd.
Picture based on research by Christopher Hoffman, Alexander Holroyd and Yuval Peres.

https://icerm.brown.edu/programs/sp-f19/

Tools and applications

Example

The (-2, 3, 7) pretzel knot



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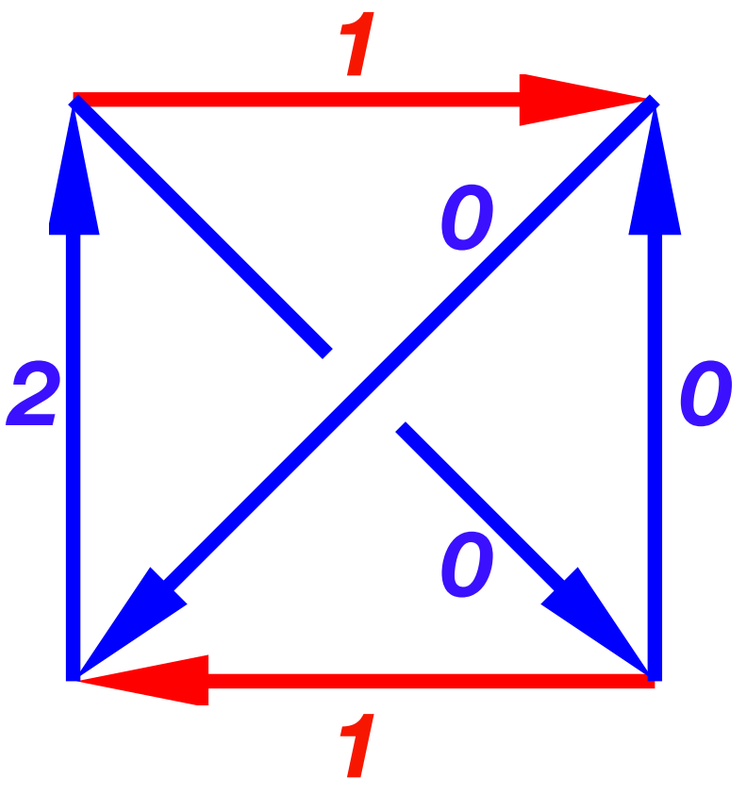
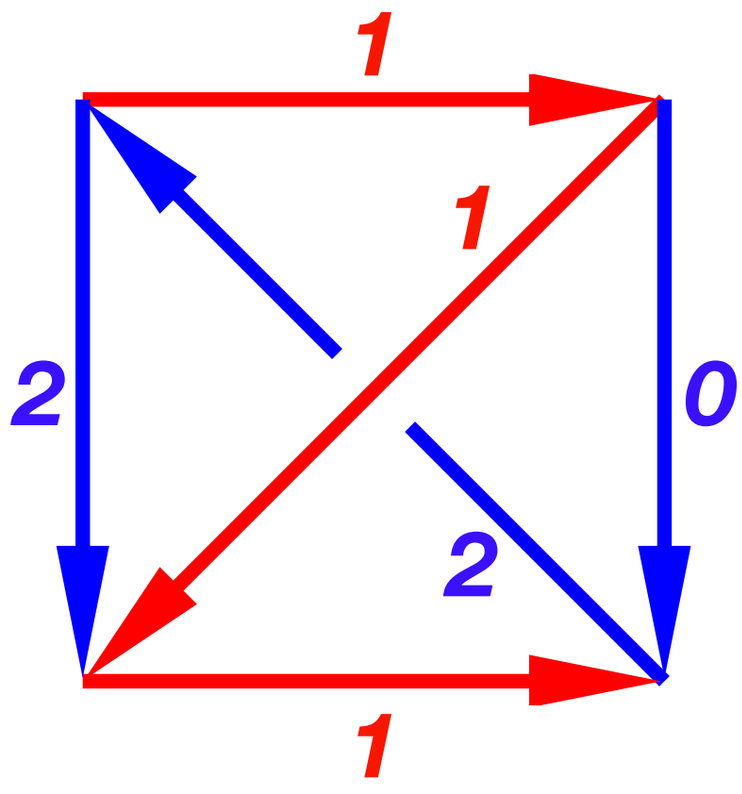
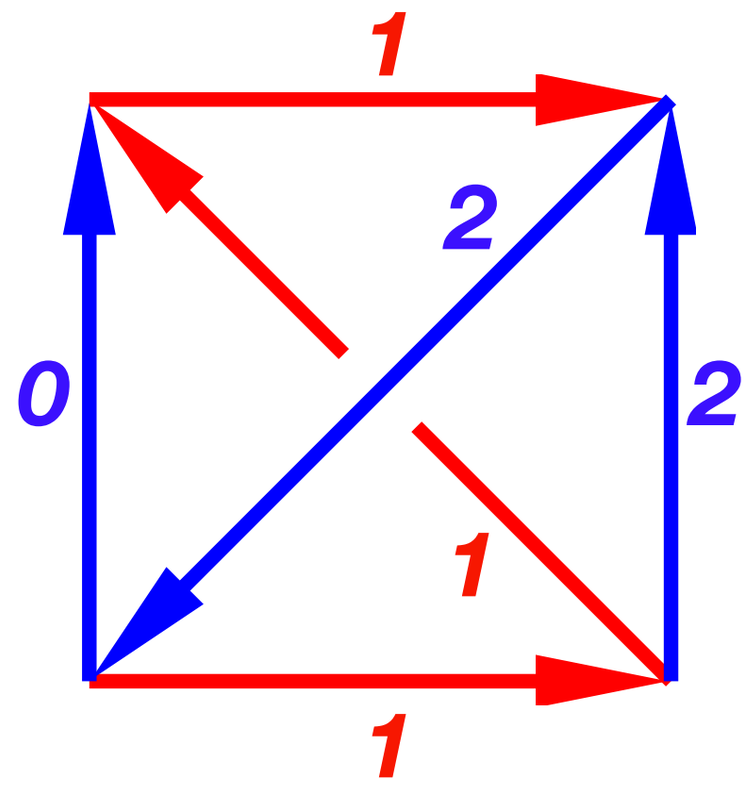
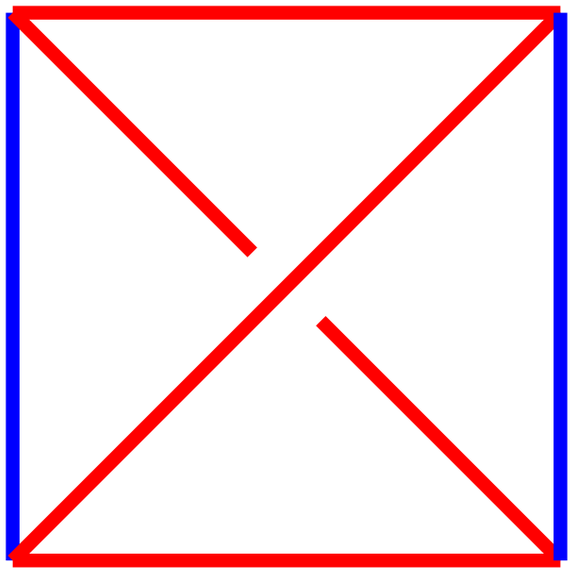




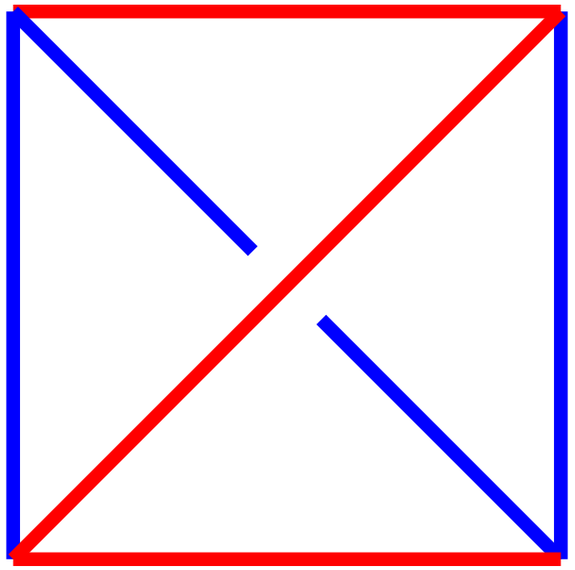


Triangulations

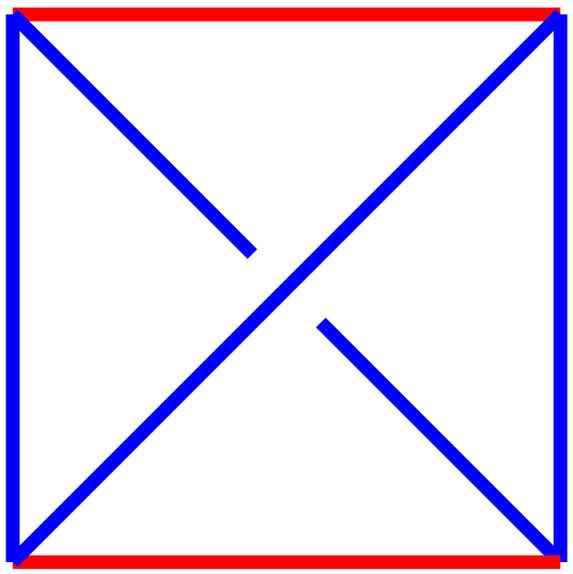
Veering tetrahedra



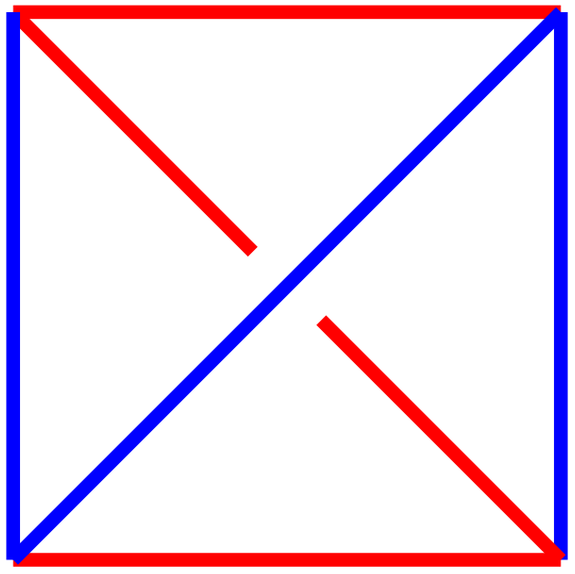
red fan



red on top
toggle



blue fan



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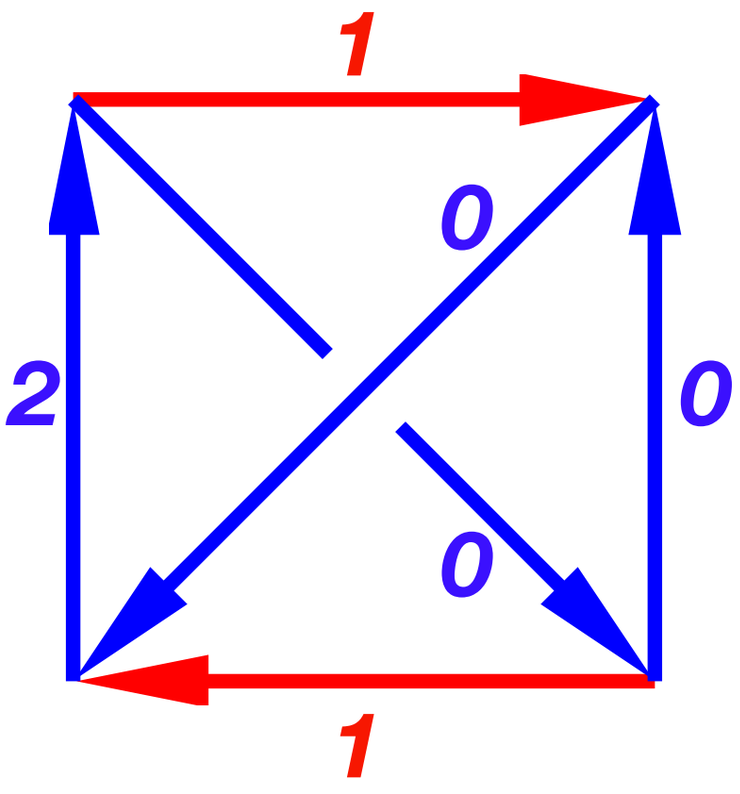
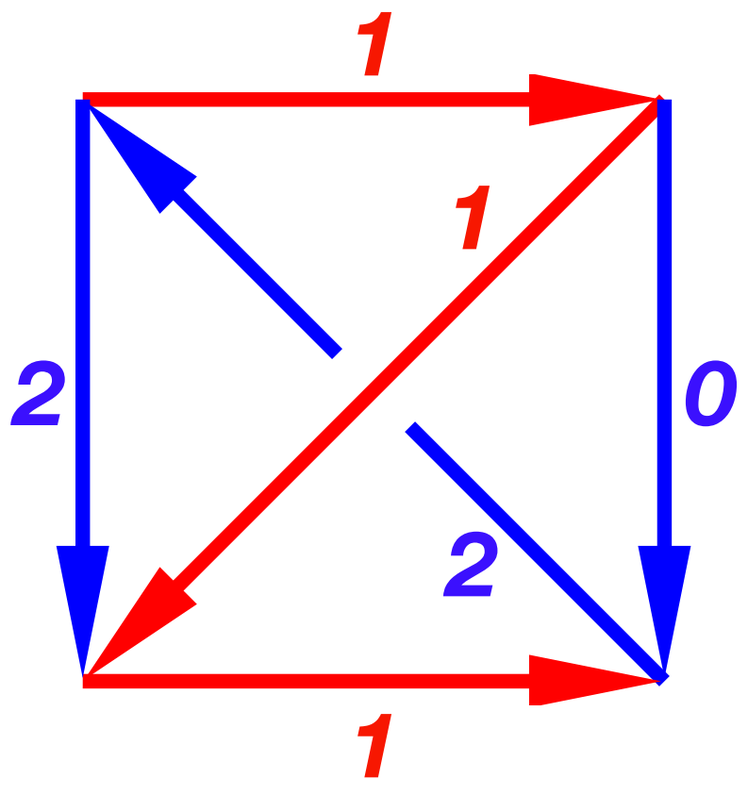
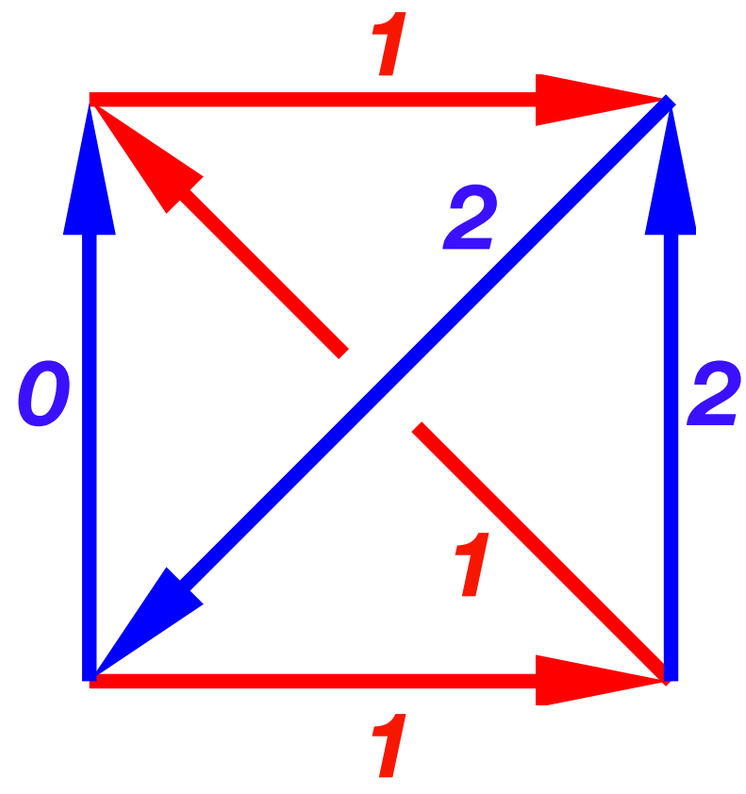
The (-2, 3, 7) pretzel knot







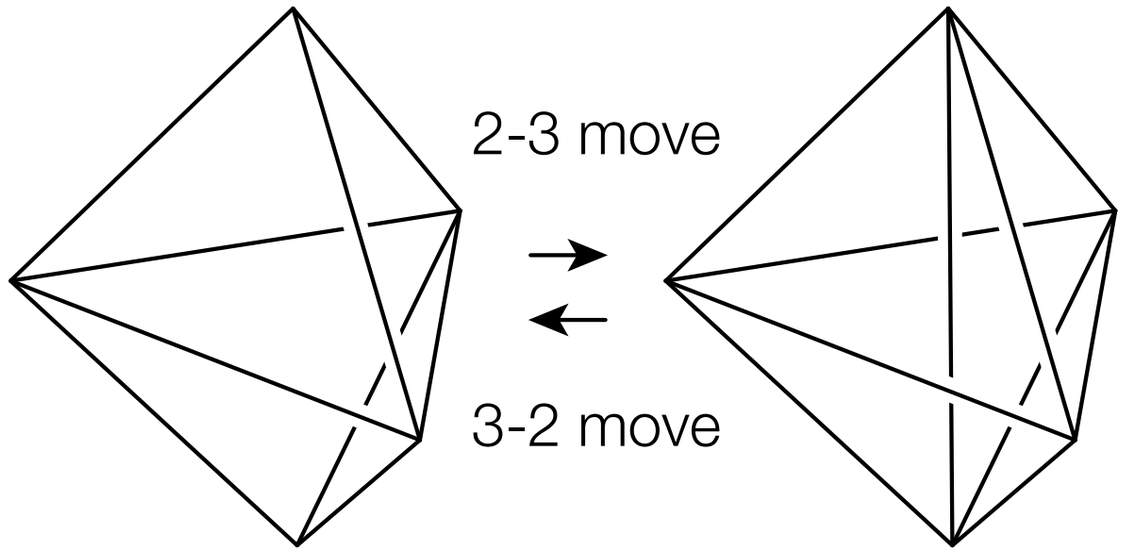
Veering triangulations are rare

The **`SnapPea`** census (up to seven tetrahedra)

- 4,815 orientable triangulations
- All are geometric so all have strict angle structures
- 13,599 taut angle structures on these triangulations
- 158 veering structures (on 151 triangulations)

Another way to sample triangulations: explore the *Pachner graph* of triangulations of a manifold.

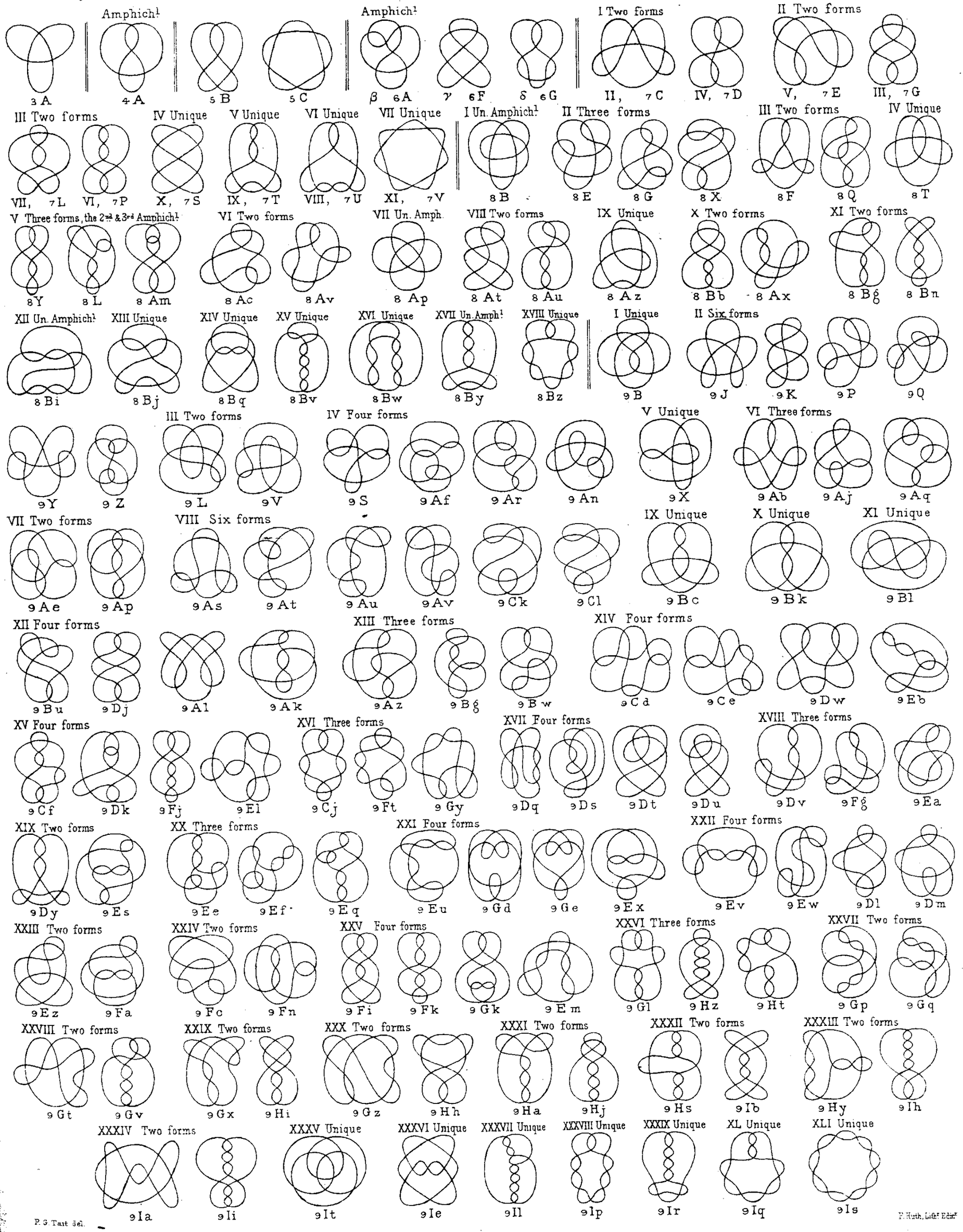
(Matveev (1987), Piergallini (1988)) The Pachner graph is connected under 2-3 and 3-2 moves.



In the “ceiling 9” subgraph of the Pachner graph for the (-2,3,7) pretzel knot complement:

triangulations	1,222,561	100%
admit a taut angle structure	153,474	12.6%
admit a strict angle structure	2,365	0.193%
admit a veering structure	1	0.0000818%

Censuses



P. G. Tait del.

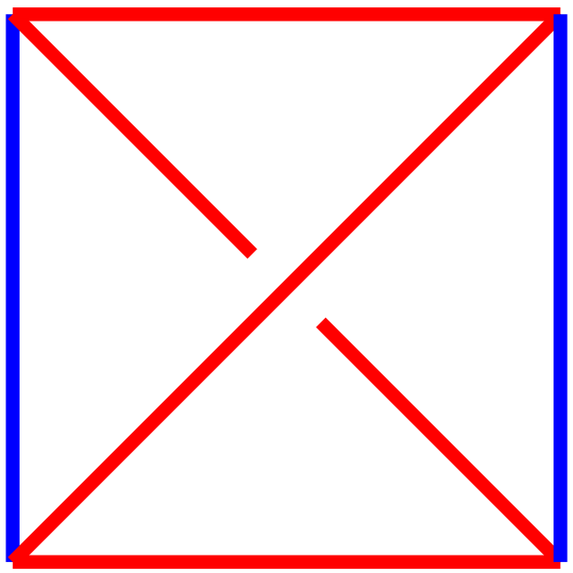
F. Huth, Lith^r Edin^r

Censuses in low-dimensional topology

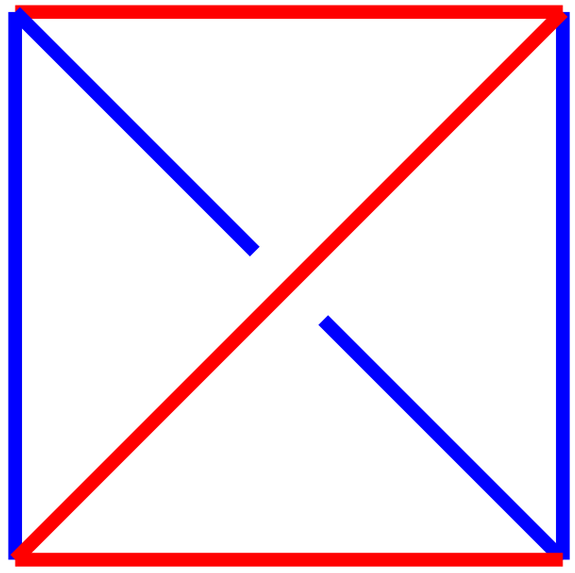
- *Knots*: Tait, Little, Conway, Rolfsen, Hoste—Thistlewaite—Weeks, Champanerkar—Kofman—Mullen, …
- *Manifolds*: Weeks, Matveev, Callahan—Hildebrand—Weeks, Thistlewaite, Burton, …
- *Triangulations of* S^3: Burton
- *Monodromies*: Bell-Hall-S, Bell

The veering census

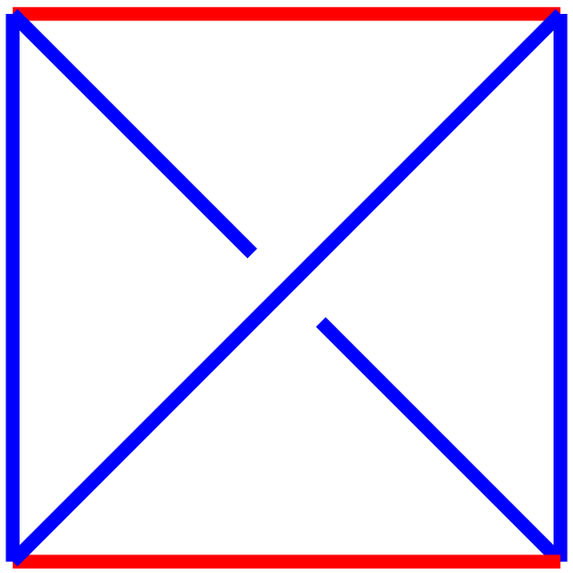
Ideal solid tori



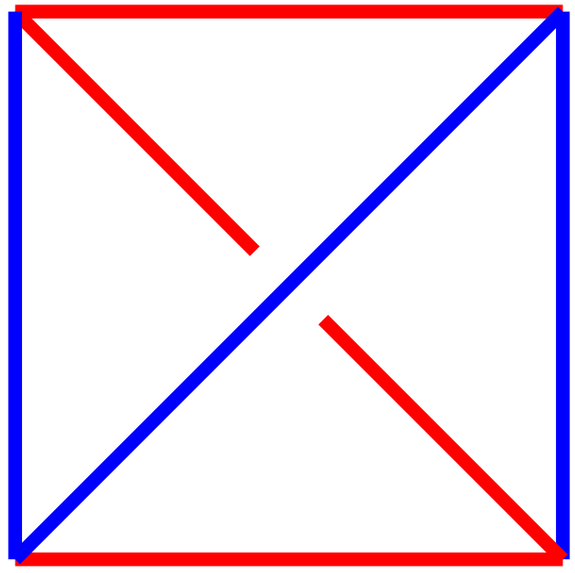
red fan



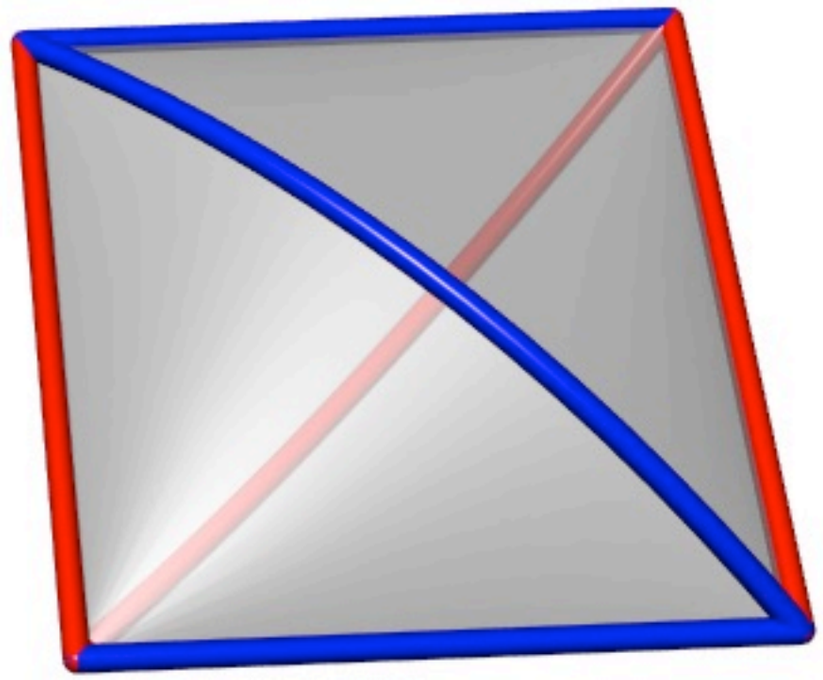
red on top toggle

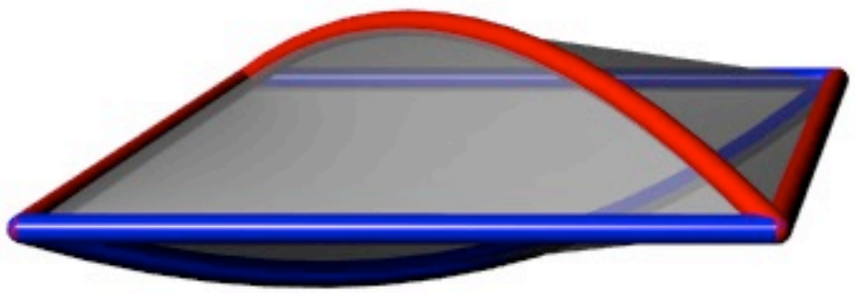


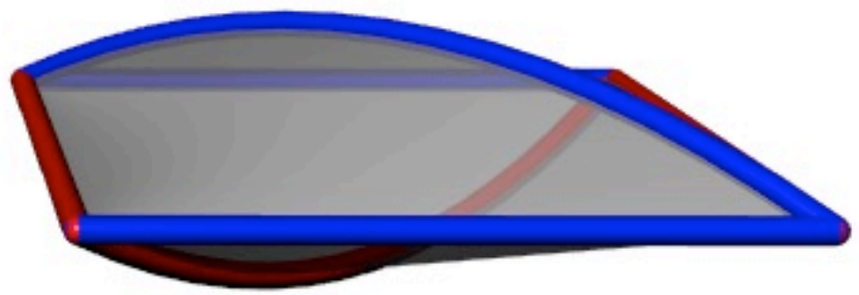
blue fan

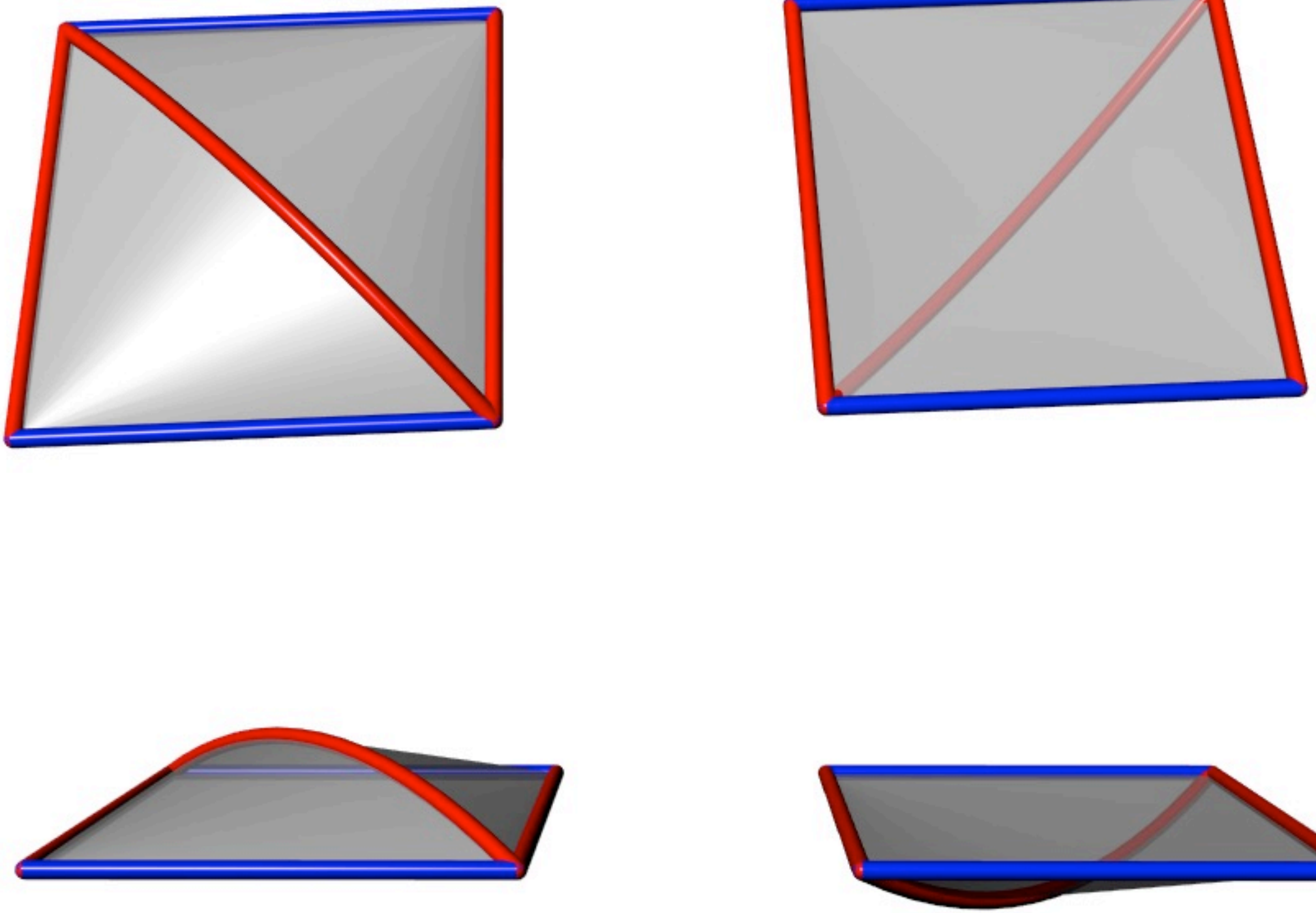


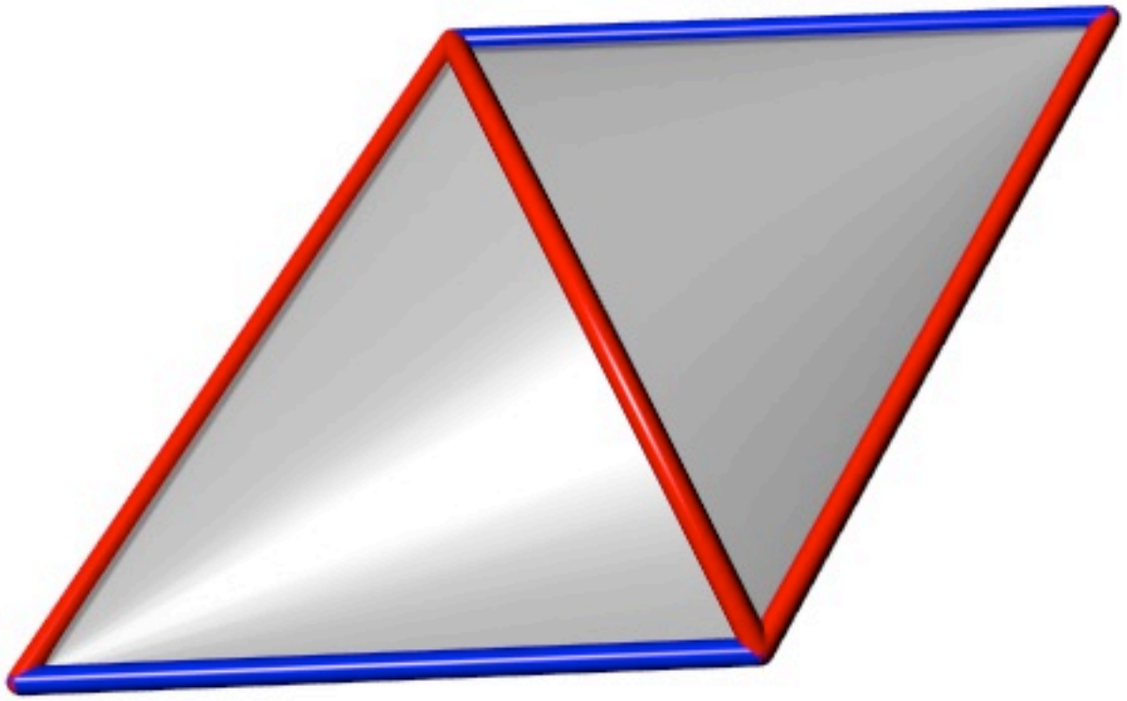
blue on top toggle

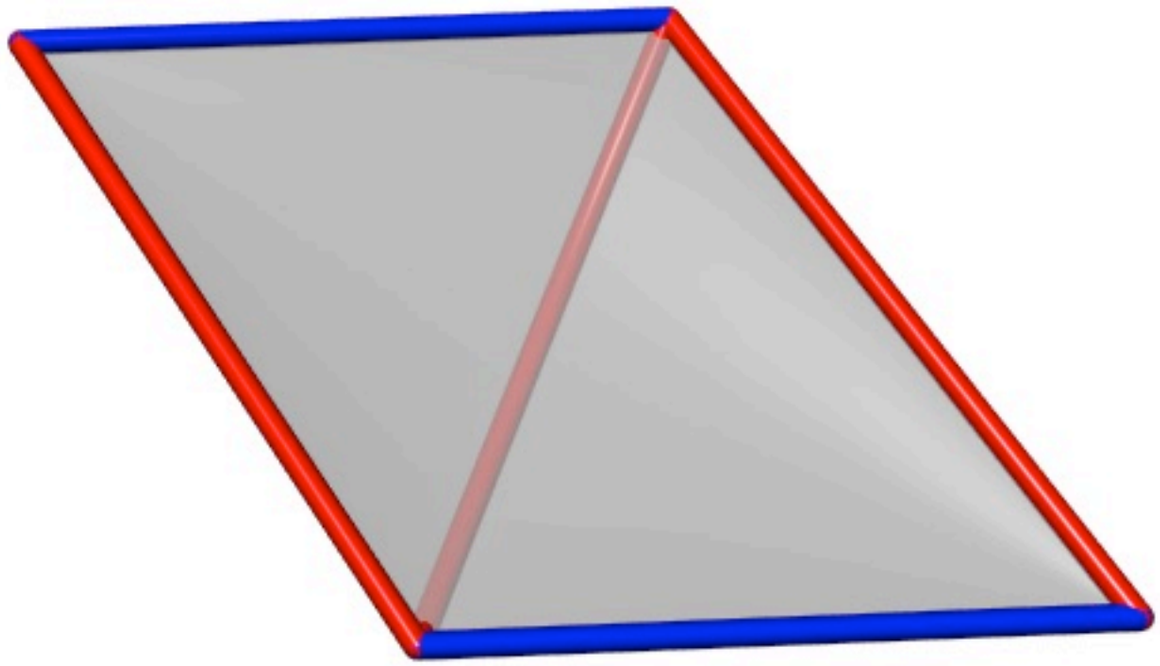


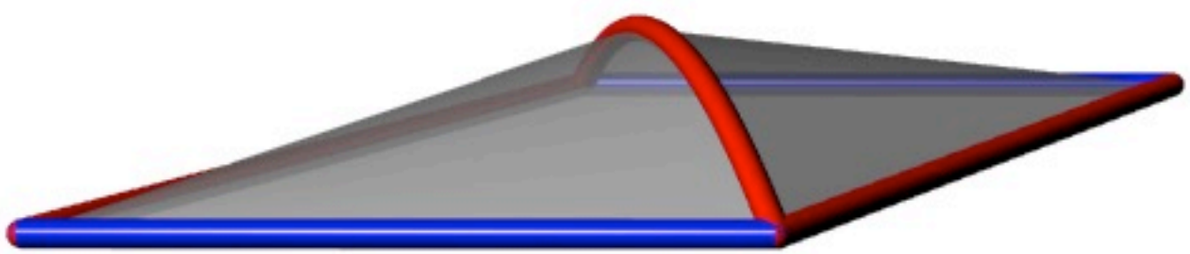


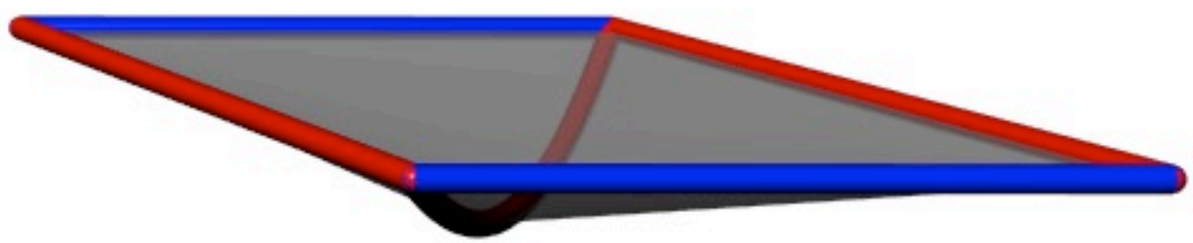


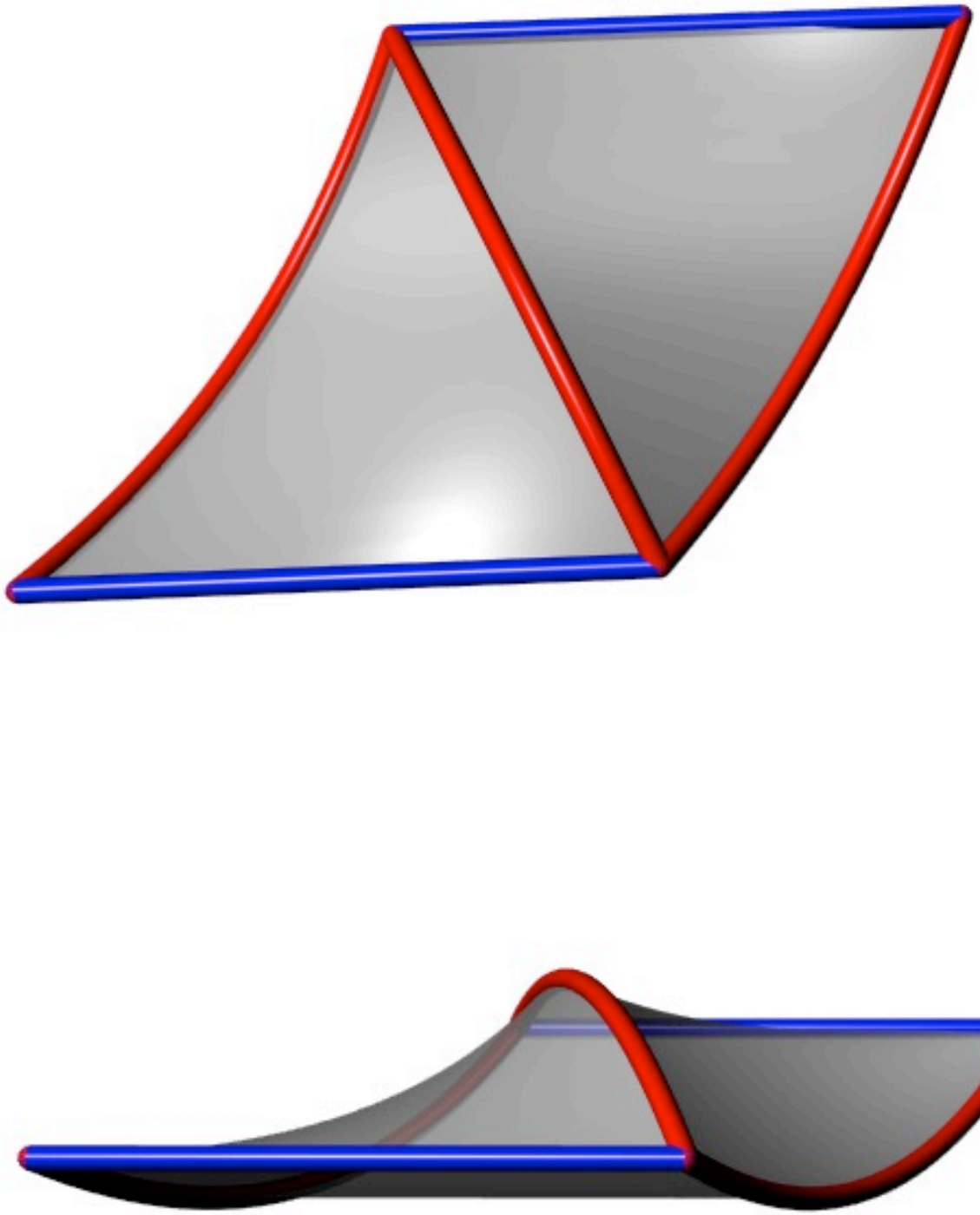


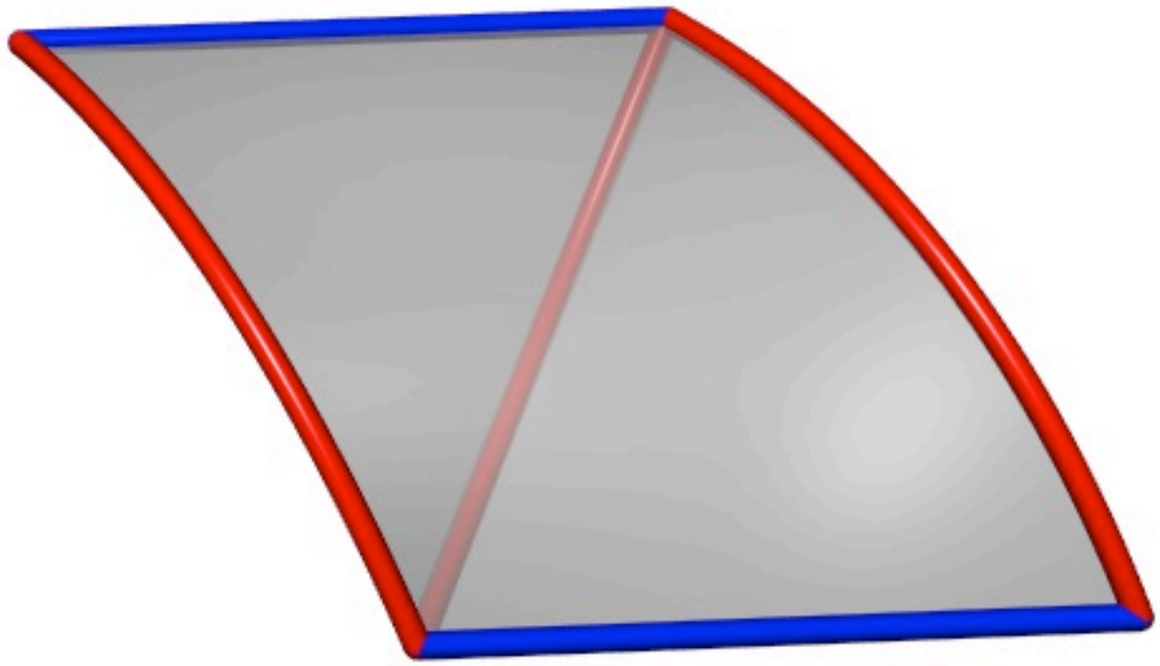


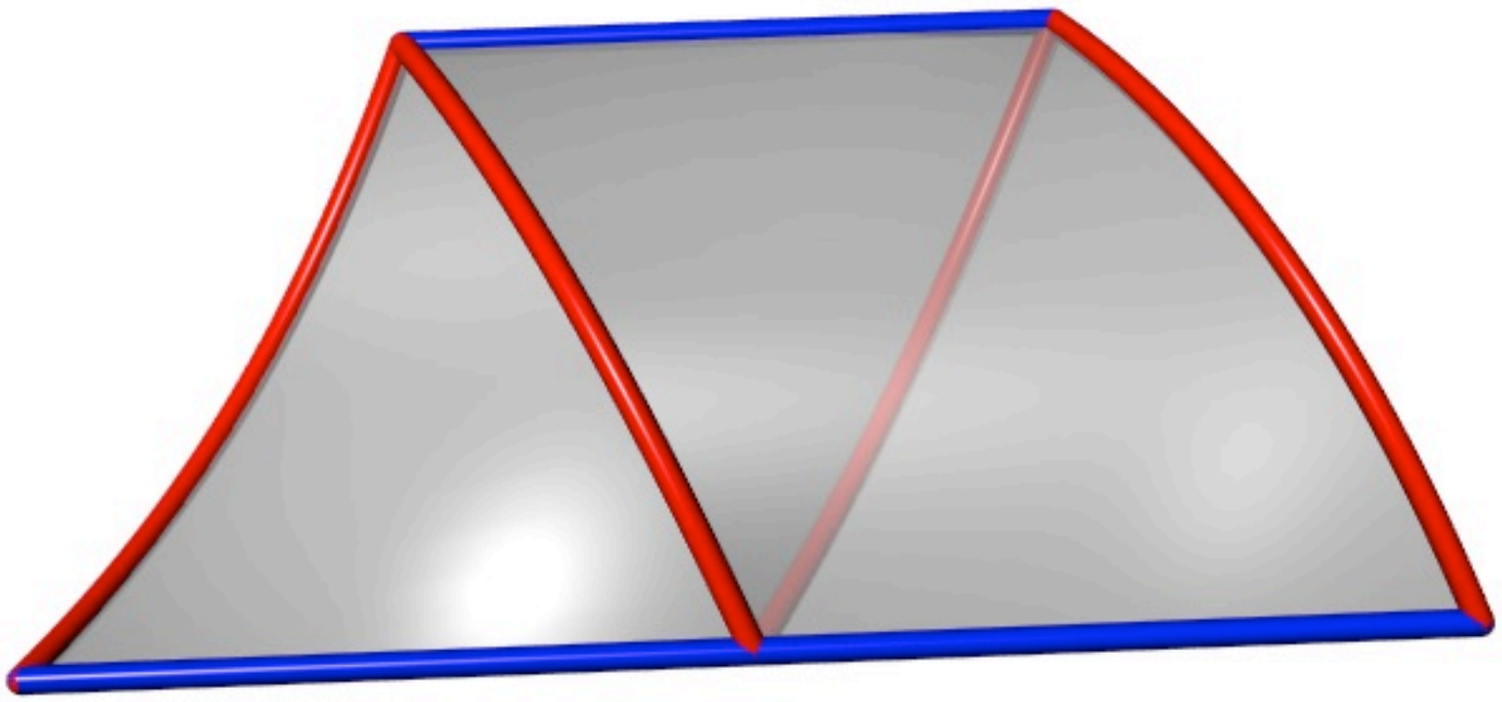


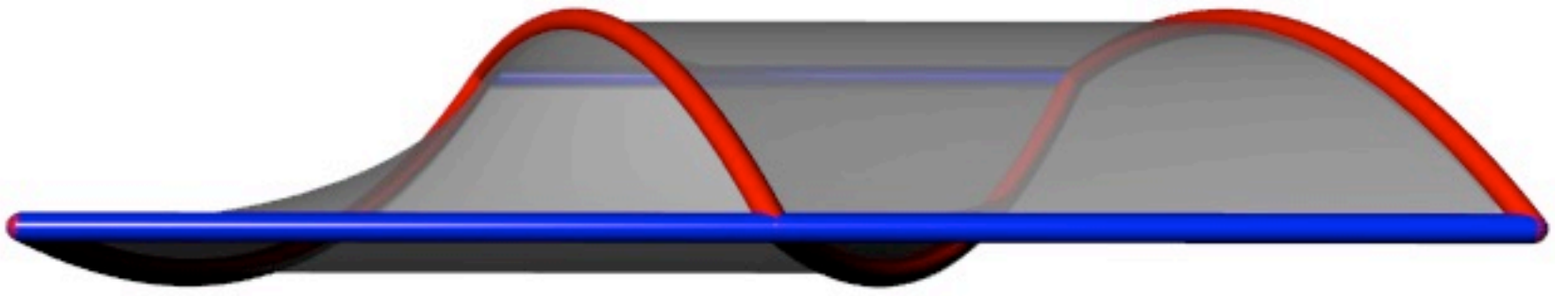


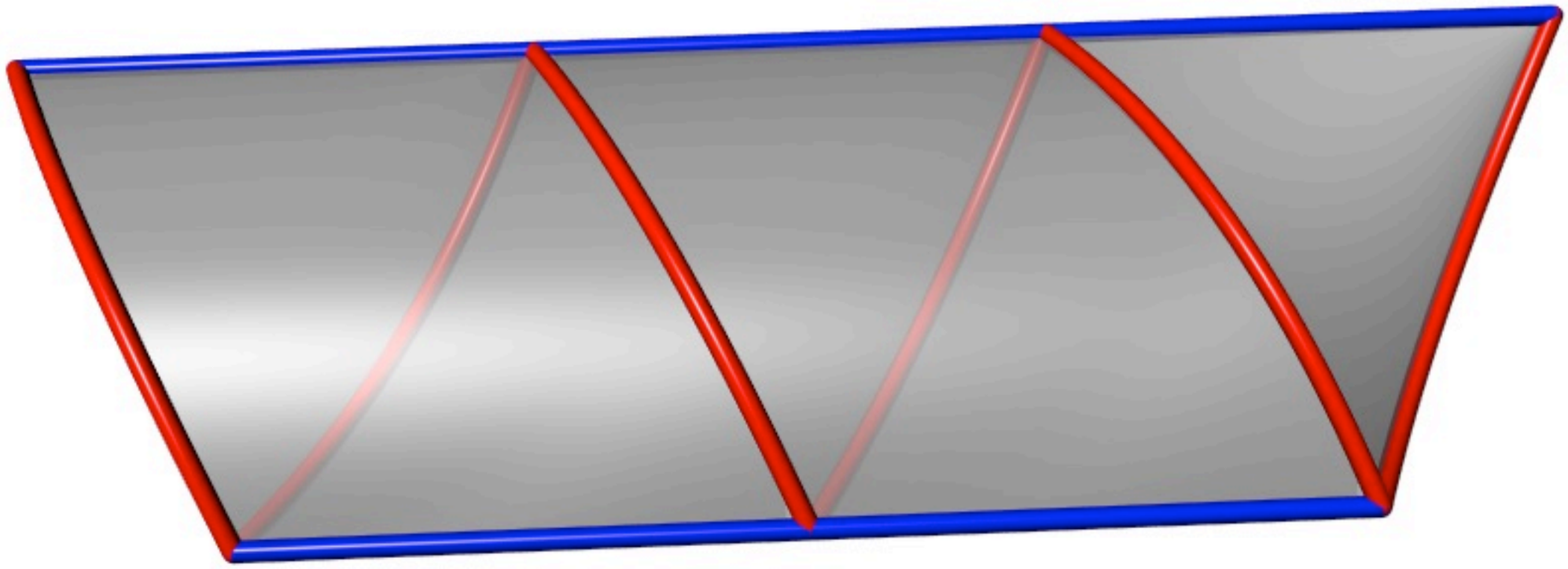


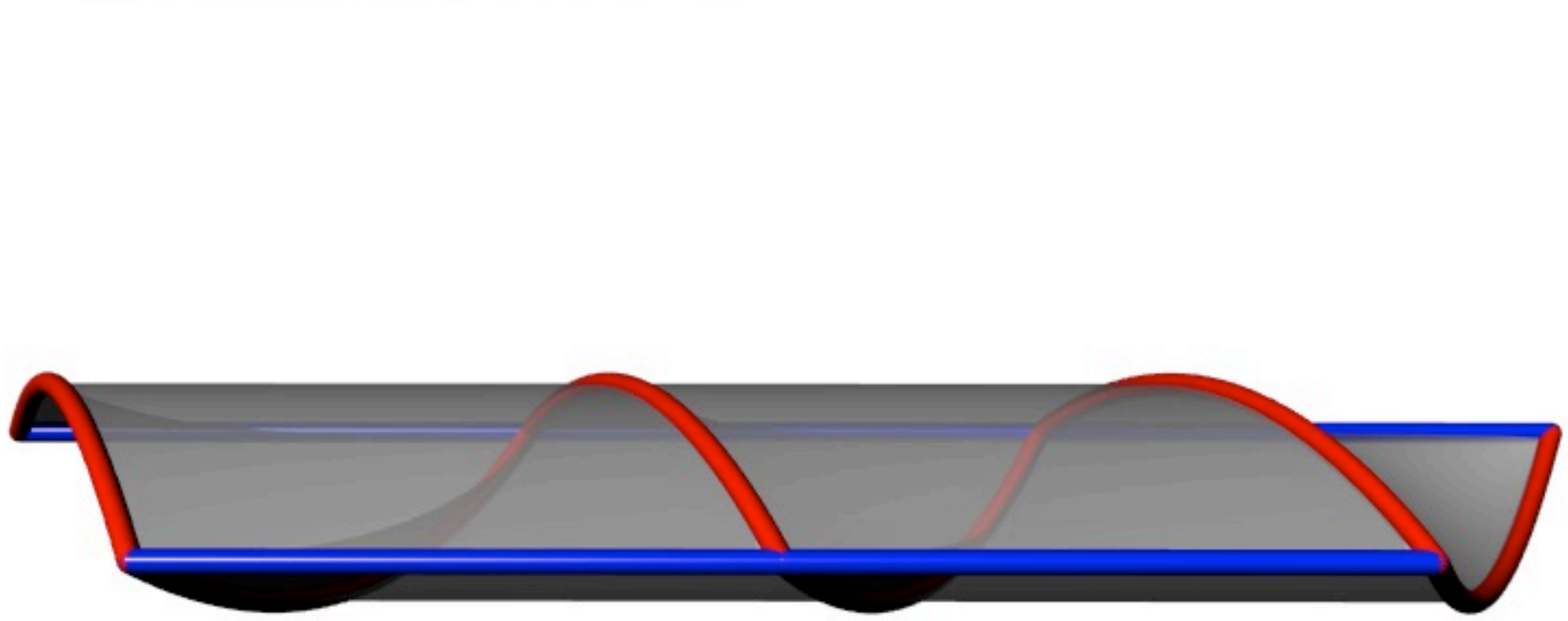




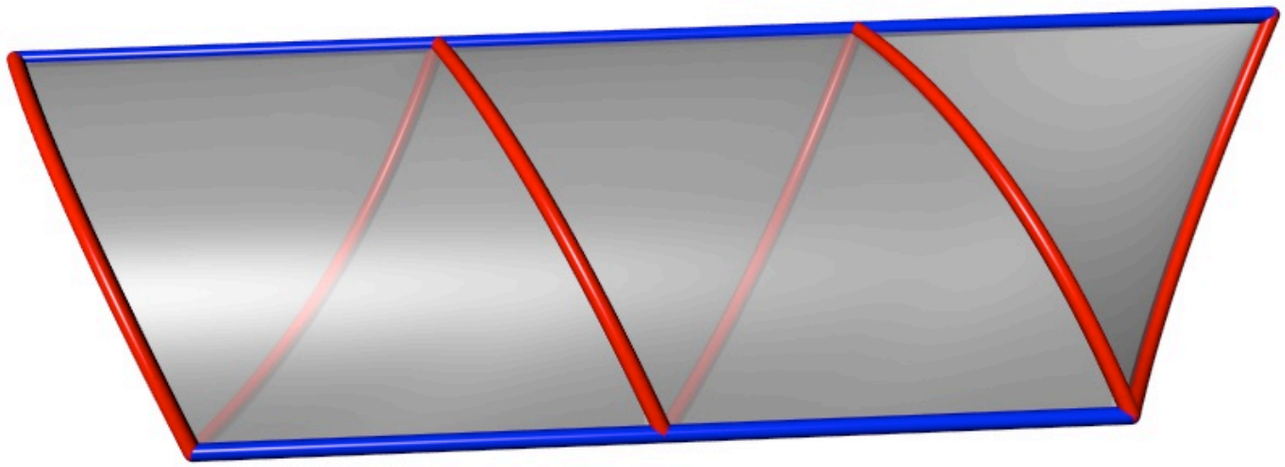




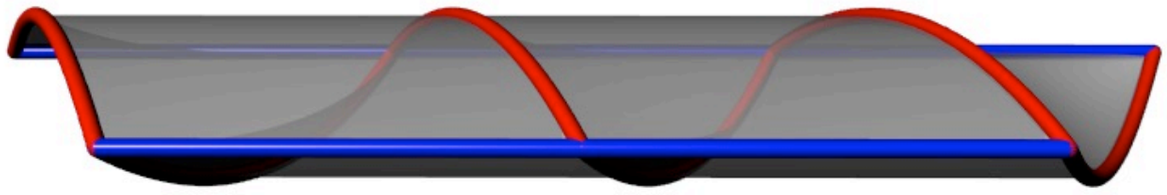




Solid tori glue to each other along rhombuses on their boundaries, matching edge colours.

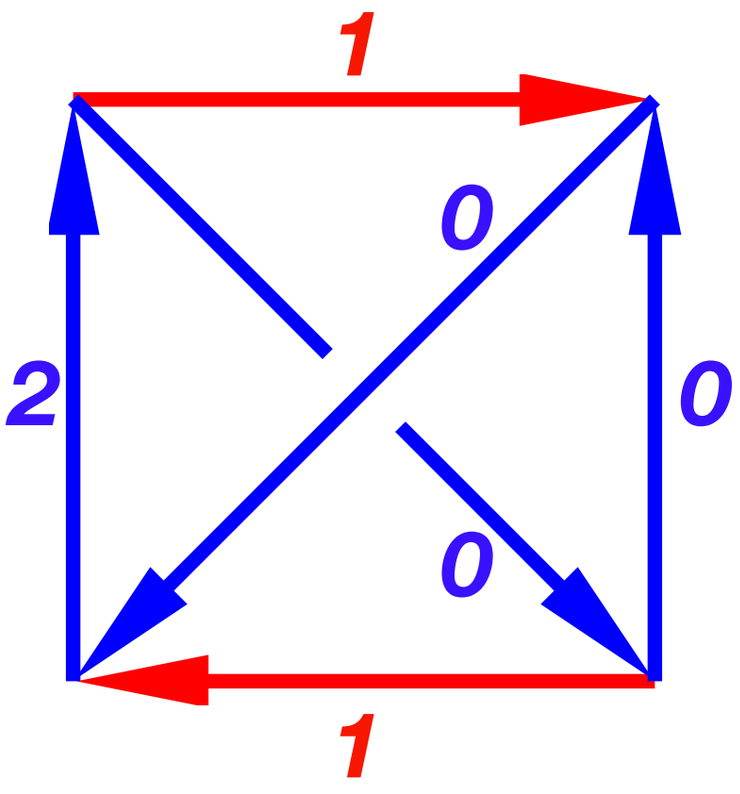


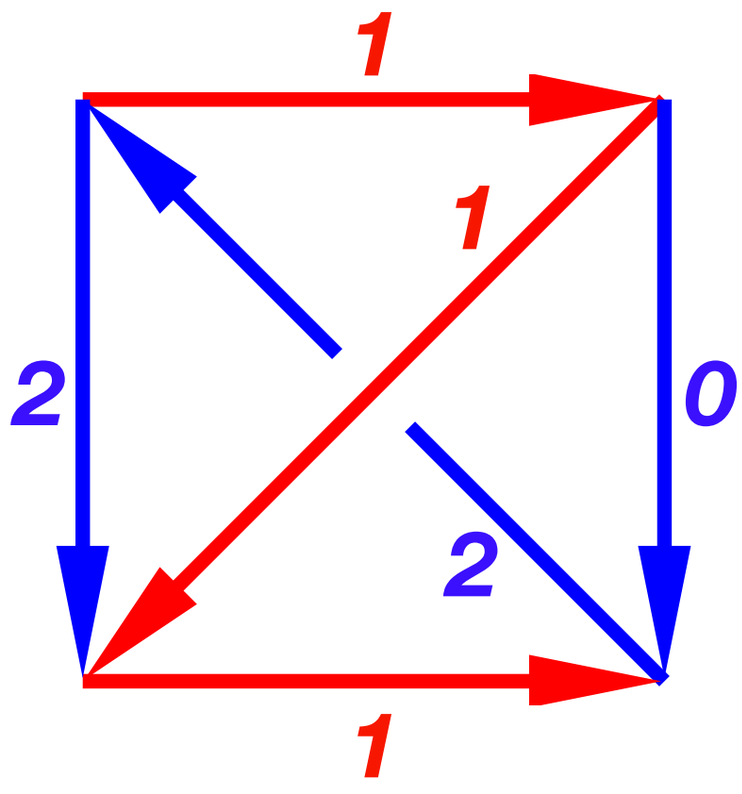
To build our census of transverse veering structures, we try all such gluings.

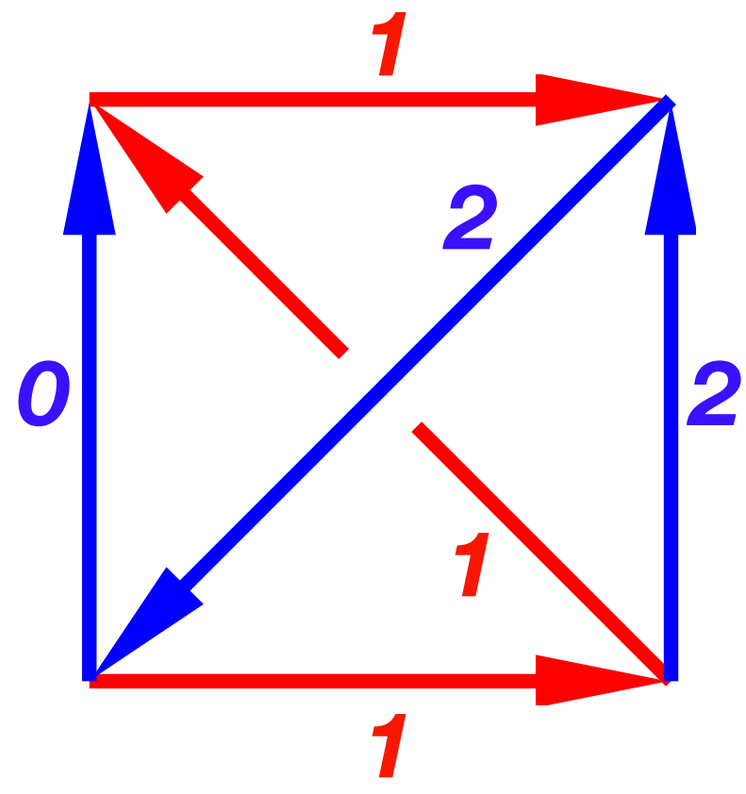


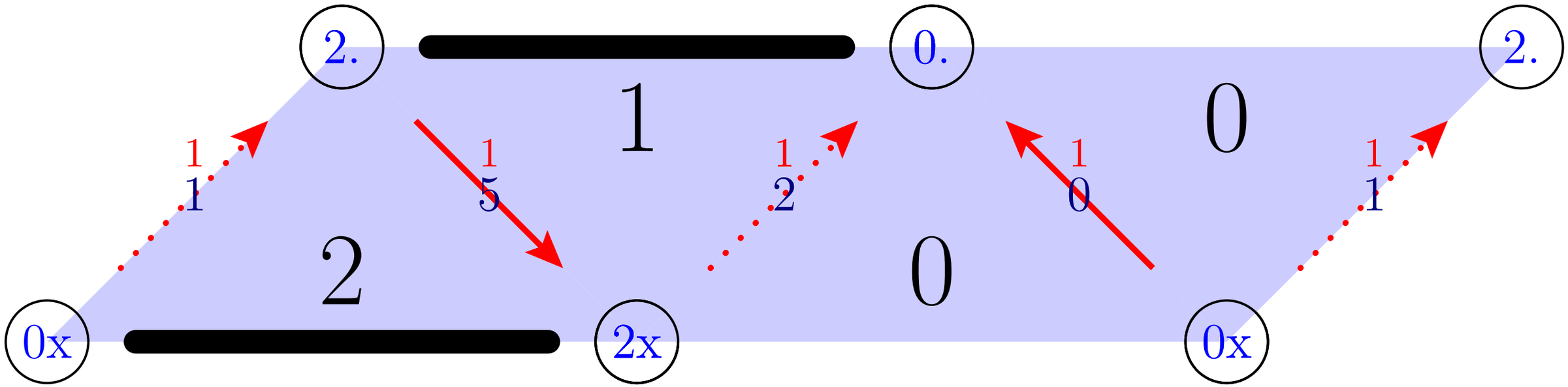
We get a transverse veering structure if the total angle at each edge is 2π.

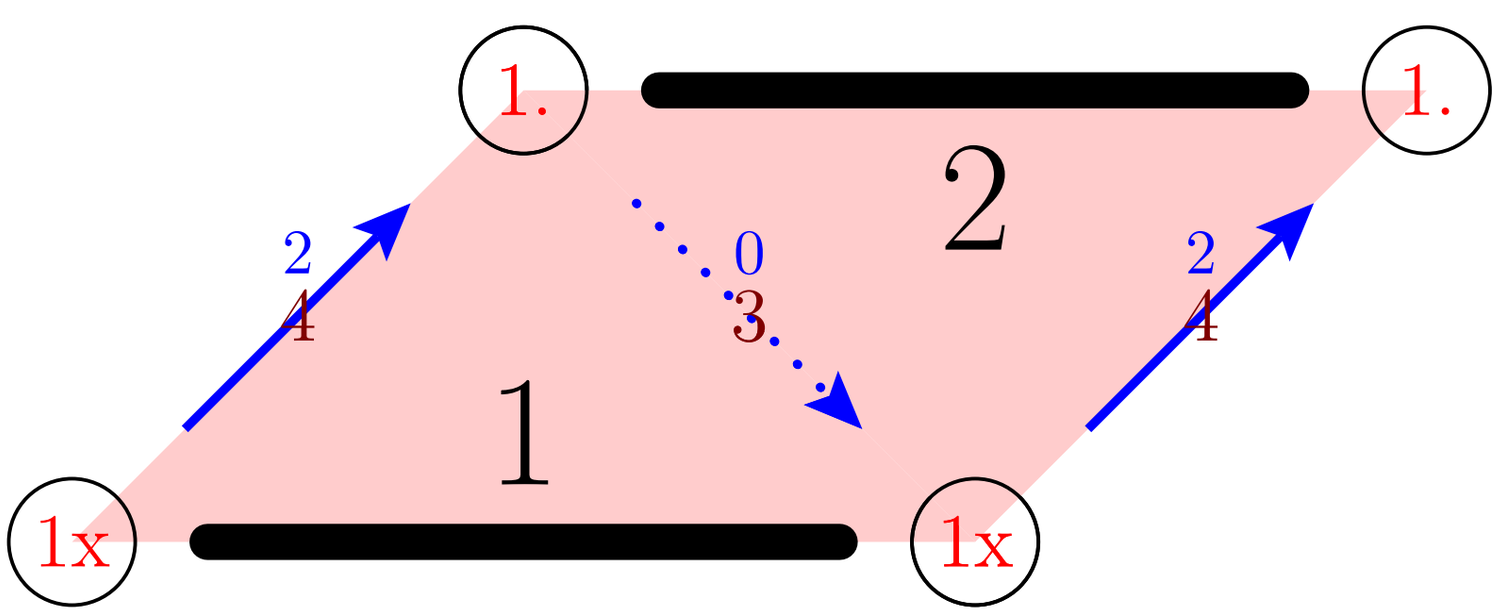
The (-2, 3, 7) pretzel knot



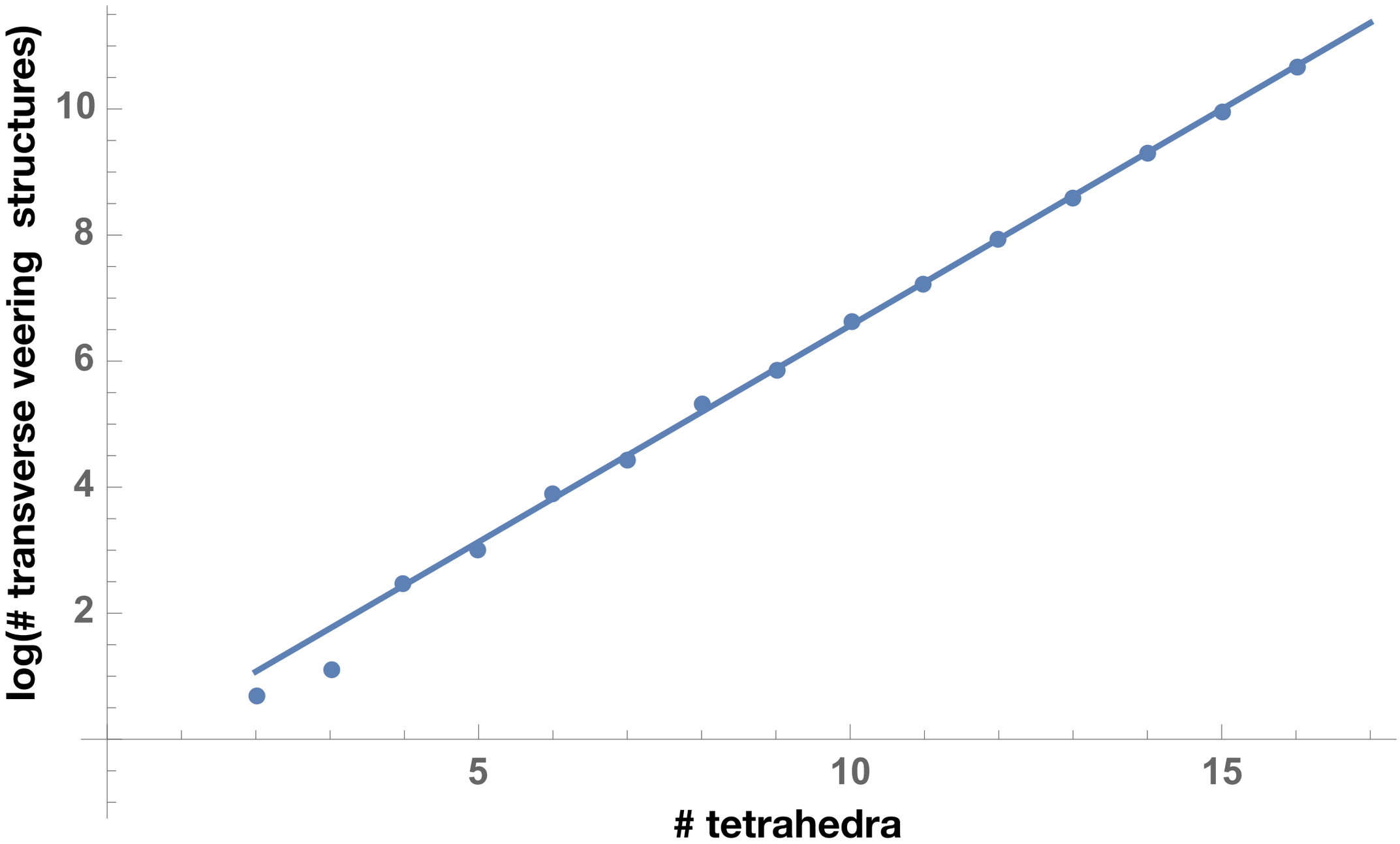





2.
0.
2.
1
0
1
1
1
5
1
2
1
0
1
1
2
0
0x
2x
0x


1.
1.
2
2
0
3.
2
4
1
1x
1x

The veering census



The number of veering structures approximately doubles every time we increase the number of tetrahedra by one.

The veering census

tetrahedra	veering	non-geometric	non-layered
2	2	0	0
3	3	0	0
4	12	0	0
5	20	0	4
6	50	0	13
7	85	0	24
8	202	0	60
9	355	1	120
10	745	3	253
11	1358	9	492
12	2867	22	1034
13	5330	52	2075
14	10972	110	4263
15	21283	234	8786
16	43763	503	18157

Census available at https://math.okstate.edu/people/segerman/veering.html

The veering census

Conjectures:

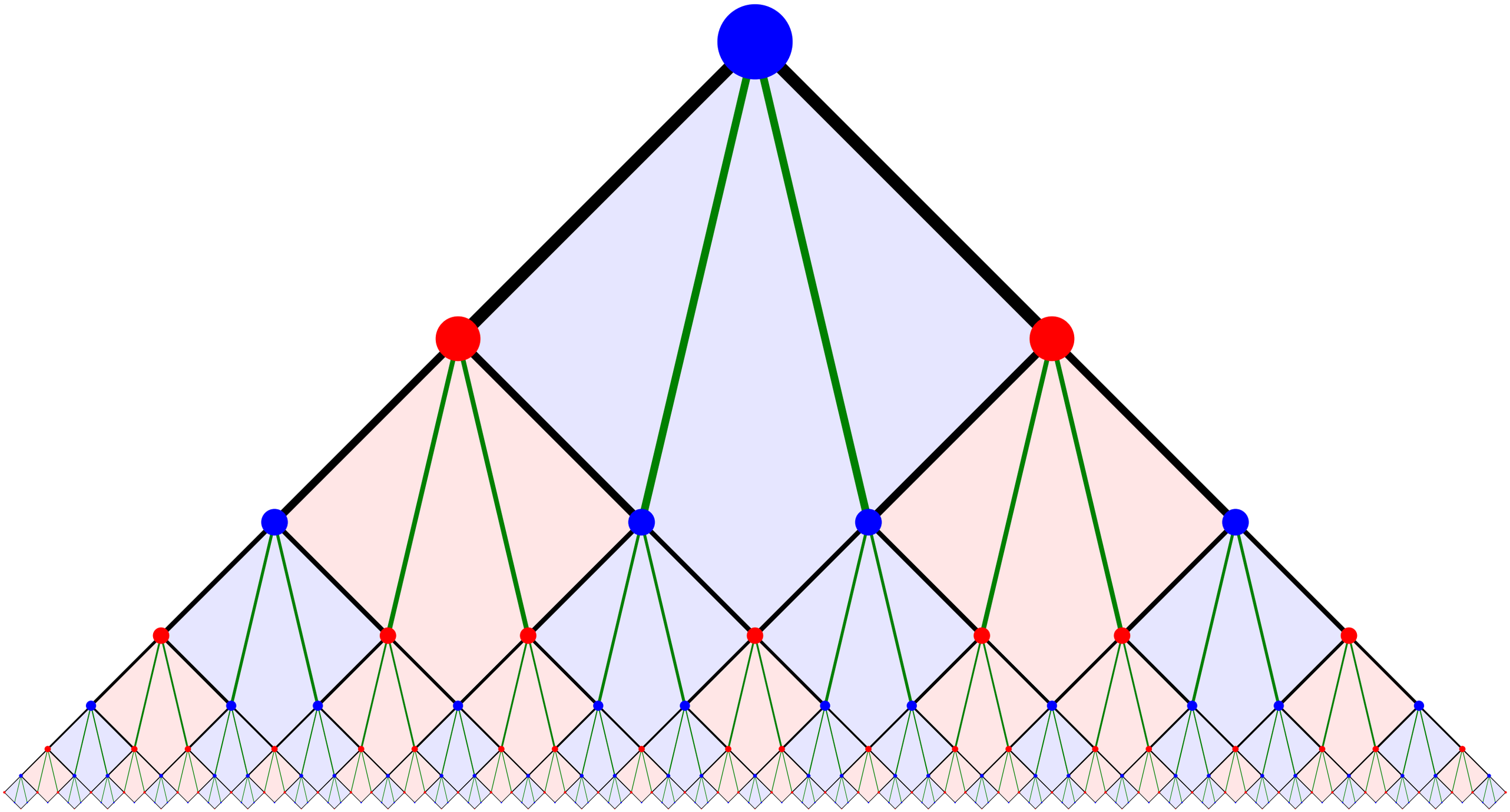
- The number of veering triangulations grows super-exponentially with n.

- The percentage of veering triangulations that are geometric tends to zero as n tends to infinity.

- The percentage of veering triangulations that are layered tends to zero as n tends to infinity.

- Any hyperbolic cusped three-manifold admits only finitely many veering triangulations (and some have none).

Thank you!



A leaf carried by the stable branched surface for the veering triangulation of the figure 8 knot complement. The leaf is decomposed into sectors, and then into normal disks.